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COMMENTARY

Why bother to spatially embed EEG? Comments on Pritchard et al., *Psychophysiology*, *33*, 362–368, 1996

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Abstract

In a recent paper, Pritchard, Krieble, and Duke (*Psychophysiology*, *33*, 362–368, 1996) studied the validity of spatial embedding of electroencephalographic (EEG) data and rejected this method in favor of time-delay embedding. The present paper describes the nonlinear characterization of brain dynamics using either spatial or time-delay embedding. We discuss the arguments published in Pritchard et al. (1996) and demonstrate that the spatial embedding cannot be rejected on this basis. We also point out the limitations of both spatial and time-delay embeddings related to the spatial extension and the high-dimensional dynamics of brain activity.

Descriptors: Nonlinear dynamics, Electroencephalogram, Embedding, High dimension, Spatially extended systems

Nonlinear dynamics has introduced new methods to deal with cerebral dynamics via electroencephalogram (EEG) quantification. The first step of these methods is a procedure termed *embedding*, which is the reconstruction from observations (i.e., recorded signals) of the time evolution of the system's state (i.e., the system's dynamics). Because the system's dynamics consists of a trajectory in the system's state space, embedding procedures provide a "reconstructed trajectory," which is topologically equivalent to the trajectory in the state space. Characteristics of the reconstructed trajectory are then quantified to infer the properties of the dynamical system underlying the signals.

Studies of EEG dynamics usually use a single-channel embedding procedure termed the *time-delay method* (for examples see Jansen & Brandt, 1993). Because this method prima facie ignores the spatial extension of brain activity, several authors have proposed to use *spatial embedding* as an alternative multichannel method (e.g., Destexhe, Sepulchre, & Babloyantz, 1988; Dvorak, 1990; Pezard, Martinerie, Breton, Bourzeix, & Renault, 1994). In a recent paper, Pritchard, Krieble, and Duke (1996, p. 367) concluded that spatial embedding "does not appear to reconstruct state space dynamics accurately." The purpose of our article is to examine the validity of this conclusion.

The first part of our article is a step-by-step discussion of Pritchard et al.'s arguments and simulations. In the second part, we briefly review possible solutions to deal with the major characteristics of brain activity: spatial extension, dynamical heterogeneity, and high-dimensional dynamics.

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Time-Delay and Spatial Embedding: A Reminder

Principles of Nonlinear Analysis: Time-Delay vs. Spatial Embedding

Nonlinear analysis can be used when a system's dynamics must be characterized on the sole basis of a set of m measurements $\mathbf{X}(t) = \{x_j(t)\}$ $(j = 1, \ldots, m)$ of the system's state $\xi(t)$. $\mathbf{X}(t)$ is related to $\xi(t)$ through a measurement function H. Thus, for a system's state $\xi(t)$ defined in a k-dimensional space $\mathbb{R}^k$, H is an application from $\mathbb{R}^k$ to $\mathbb{R}^m$ and $\mathbf{X}(t) = H[\xi(t)]$. For EEG, $\mathbf{X}(t)$ corresponds to m potentials values (recorded over m electrodes), which depend on $\xi(t)$, that is, the activities of all the k macro-columns of the brain.

The goal of dynamical analysis is to infer the dynamics (i.e., the ξ-trajectory in the state space $\mathbb{R}^k$) from the measurements $\mathbf{X}(t)$. Under the assumption that the time evolution of $\xi(t)$ is governed by a differential system, this is achieved following a three-step procedure:

- A trajectory, topologically equivalent to the ξ-trajectory, is reconstructed from the measurements $\mathbf{X}$. This step is the so-called "embedding procedure."
- Topological invariants of the reconstructed trajectory are then computed, for instance: dimensions, Lyapunov exponents, nonlinear prediction, and so on (for a recent review see Kantz & Schreiber, 1997). These invariants are closely related to the dynamics.

- In addition, because correlated noise can depict similar characteristics as truly nonlinear dynamics, surrogate data testing has been introduced to validate the nonlinear indices obtained with the first two steps (e.g., Prichard & Theiler, 1994; Theiler, Eubank, Longtin, Galdrikian, & Farmer, 1992).

Measurements at each recording site j, during an epoch T at a sampling rate $1/\tau$, provide a time series $\{x_j(i.\tau)\}$ ($i = 1,\ldots,N$ and $T = N \times \tau$), which can be embedded using several methods (successive derivatives, time-delay, spatial, and spatiotemporal embeddings). In this section, we will restrict our discussion to the methods discussed in Pritchard et al. (1996). The embedding method is applied to these time series to build an observation matrix $\mathbf{V}$ defined as: $\mathbf{V} = \{\mathbf{v}\ (t)\}$ with $t = i \cdot \tau$ for $i = 1,\ldots,N_v$ and $\mathbf{v}(t)$ are n-dimensional vectors. The N_v columns of $\mathbf{V}$ are N_v successive vectors corresponding to a trajectory in an n-dimensional space.

The first technique, *time-delay embedding* (or delay method or single-channel method), uses recordings from one single site j and the vectors $\mathbf{v}(t)$ are defined as:

$$\mathbf{v}(t) = \{x_j(t); x_j(t + l \cdot \tau); x_j(t + 2 \cdot l \cdot \tau);\ldots; x_j(t + (n-1) \cdot l \cdot \tau)\}$$

with l integer.

Alternatively, the second technique, *spatial embedding* (or multichannel method), uses recordings from n sites ($n \leq m$) and the vectors $\mathbf{v}(t)$ are defined as:

$$\mathbf{v}(t) = \{x_1(t); x_2(t);\ldots; x_n(t)\}.$$

Those procedures are based on two fundamental theorems: time-delay embedding is based on Takens' theorem (Takens, 1981) and spatial embedding on Whitney's theorem (Whitney, 1936). Those theorems have been studied and generalized by Sauer, York, and Casdagli (1991).

Rejection of Pritchard et al.'s Arguments

On the basis of the above definitions, we will comment on the main arguments that led Pritchard et al. (1996) to reject spatial embedding in favor of the time-delay method. Although the authors provided a correct definition of spatial embedding, several points in their discussions of "the issue of stationarity," "the origin of spatial embedding," and "differentiable embedding," need to be clarified.

The issue of stationarity. Both embedding methods only apply to stationary dynamics, which means that during the period of observation, the system's parameters should remain constant in time. Analogously with this constraint of time-stationarity for temporal embedding, Pritchard et al. (1996, p. 363) introduced, in the case of spatial embedding, the new constraint of "spatial stationarity" in the sense of a spatially homogeneous dynamical behavior. Contrary to the temporal stationarity, this constraint is not actually needed for the application of the spatial embedding. Furthermore, a spatially extended system can split into domains depicting different dynamical behaviors while following a time stationary dynamics (Kaneko, 1989). The fact that "the human cortex has regional neuropsychological specialization" should be related to this *spatial dynamical heterogeneity* and should be treated specifically.

The origin of spatial embedding. In a seminal paper, Eckmann and Ruelle (1985, p. 627) proposed that spatial embedding may be used for the reconstruction of the dynamics from an experimental signal and concluded that "of course one should measure several experimental signals instead of only one whenever possible." This approach was applied to deal with the problem of "spatially localized degrees of freedom" (pp. 648–649, the very paragraph to which Pritchard et al. referred). They did recommend multiple simultaneous recordings of the same system's state to obtain characteristics regarding the whole system (which is markedly different from "the recording at a given locus from a set of identical brains [i.e., systems] in the same dynamical state" in Pritchard et al., 1996, p. 363).

Differentiable embedding. Pritchard et al. (1996) conjectured inadequately that spatial embedding should not be differentiable (p. 363) because Whitney (1936) and Takens (1981) proved that both reconstruction methods lead to a differentiable embedding (Ott, Sauer, & Yorke, 1994). Moreover, EEG can be approximated by a linear combination of brain state variables (Lachaux et al., 1997) that is indeed continuous and differentiable.

Inadequate simulation. Pritchard et al. (1996) used a "Lorenz system" simulation (pp. 363–365) as their key argument to conclude that "as a reconstructor of state-space dynamics, it [spatial embedding] fails." The authors produced three time series $\{x(t), y(t), z(t)\}$ governed by Lorenz differential equations. The observation matrix $\mathbf{V} = \{\mathbf{v}(i)\}$ is constructed from the time series $x(t)$ with ($i = 1,\ldots,1024$ and $n \leq 15$):

$$\mathbf{v}(i) = \{x(i); x(i + 1024); x(i + 2 \times 1024);\ldots; x(i + (n-1) \times 1024)\}.$$

This procedure is not a spatial embedding but a time-delay method with a long time window ($w = (n-1) \times 1024$). It is thus ill-suited to prove anything about spatial reconstruction. Pritchard et al. (1996, p. 363) argued that their simulation mimics a spatial embedding because "each series had the same chaotic dynamics, that is, each was 'on the same attractor.'" Nevertheless, in case of spatial embedding, the n coordinates of the reconstructed vectors must be n simultaneous measurements of the same state point and not of the same attractor. In their simulation, increasing the embedding dimension leads to the addition of a time series measuring a system of dimension 2.07: no saturation can be expected in this case.

Several numerical tests dealing with the problem of spatial embedding of brain dynamics have been proposed (Lachaux et al., 1997) on the basis of a simulated EEG. These tests have shown that the time-delay reconstruction was ill-suited for the reconstruction of spatially extended systems, whereas spatial embedding performed better.

Pritchard et al. (1996) showed that dimension estimation using multichannel embedding may be fooled by linear Gaussian processes with cross-channel correlation. But, in this case, an appropriate surrogate data test (Prichard & Theiler, 1994) does not permit one to reject the linearity hypothesis. In the case of cross-correlated signals, singular value decomposition should be used to obtain independent coordinates for spatial embedding.

Conclusion

From the above discussion we conclude that the theoretical arguments do not reject spatial embedding. The rejection of spatial embedding by Pritchard et al. (1996) arises from the following confusion: EEG spatial embedding is based on the simultaneous measurements of the system's state and not on the measurements of one state variable of the same attractor.

Dealing With Brain Dynamics

High Dimension of Brain Dynamics

Cerebral cortex is composed of approximately 10^4 or 10^5 macrocolumns the activities of which are recorded through EEG (Nunez, 1990). This number gives an approximate lower bound for the state space dimension of macroscopic brain dynamics. However, due to delays and the long range interactions in neuronal systems, brain dynamics should be more reliably defined in an infinite dimensional space. Nonlinear analysis relies on the fact that most infinite dimensional systems explore a subset of their state space in which finite dimension is the critical parameter for embedding techniques (Sauer et al., 1991). However, finite dimension does not mean low in the sense of embedding dimensions usually used (less than 10) and neither spatial embedding nor time-delay permits one to correctly describe high-dimensional dynamics (Lachaux et al., 1997).

Sauer et al. (1991) proposed theorems, for time-delay and spatial embedding, dealing with partially incorrect reconstructions due to a too low embedding dimension (i.e., $n < 2d$). These theorems are of little help in the case of an experimental procedure because they suppose that the subset's dimension is known. However, it has been shown that correlation dimension (Ding, Grebogi, Ott, Sauer, & Yorke, 1993) and prediction (Schroer, Sauer, Ott, & Yorke, 1998) can be computed with a certain accuracy even when the trajectory is partially reconstructed.

The most obvious way of dealing with high-dimensional dynamics is to increase the embedding dimension, which can be achieved in several ways:

- In the case of time-delay embedding, the number of time-delayed coordinates can be increased. Nevertheless, the important loss of correlation between the first and the last coordinates is a drastic drawback of this solution (Martinerie, Albano, Mees, & Rapp, 1992).
- In the case of spatial embedding, the number of recording sites can be increased. This solution is limited in the case of EEG, because the correlation between recording sites entails that new sites do not increase significantly the quantity of information about brain dynamics. The redundancy between the recordings can be reduced using singular value decomposition (SVD) of the observation matrix, without altering the characteristics of the dynamics (Albano, Muench, Schwartz, Mees, & Rapp, 1988).
- Both spatial and time-delay methods can be used (Kantz & Schreiber, 1997) following a generalization of Takens' theorem (Sauer et al., 1991). An example of its application to simulated signals is depicted on Figure 1. This figure shows that the correlation dimension computed in the case of spatial embedding with time delay underestimates the real correlation dimension more than spatial embedding. This method has been used with success for the analysis of neuroelectrical data (Martinerie et al., 1998).

Nevertheless, increasing the embedding dimension still suffers from limitations including computation cost, loss of simple geometrical representation and increase of noise level.

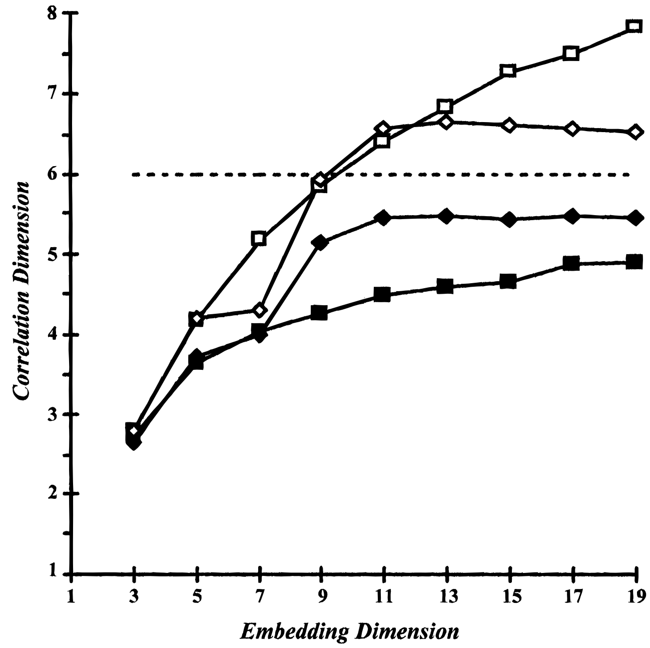

Figure 1. Correlation dimensions computed for a coupled map lattice system, measured through a three-shell spherical model, as a function of the embedding dimension in two reconstruction cases: using the *multichannel method* with the measurements (♦ for raw data and ◇ for the mean value of surrogate data), and using the *multichannel method with time delay* for four measurements (■ for raw data and □ for the mean value of surrogate data). The dotted line depicts the correlation dimension computed within the 13-dimensional state space with the original data (5.98). The standard deviation observed for the surrogate data cannot be depicted on the figure. The differences between raw and surrogate data correlation dimensions are always significant.

These data were computed with the method used in Lachaux et al. (1997). A simulated electroencephalogram was generated, using direct propagating equations, on 19 electrodes placed on the scalp (according to the 10-20 international electrode placement system) of a three-shell spherical model of the head. The activities over time of 13 cortical current dipoles were simulated according to the following definition of coupled map lattice:

$$x_i(t) = (1-\epsilon) \times F[x_i(t)] + (\epsilon/2) \times \{F[x_{i-1}(t)] + F[x_{i+1}(t)]\}$$
$$F(x) = 1 - a \cdot x^2$$

for $i = 1, \ldots, 13$; $a = 1.9$; $\epsilon = 0.3$ and periodic boundaries.

Cerebral Cortex as a Spatially Extended System

Connections between different cortical areas can be described at different levels (Schüz, 1995) as short range (local projection of pyramidal cells), medium range (axonal projection of pyramidal cells from a sulcus to neighboring ones), and long range connections (such as corpus callosum or telencephalic fasciculi). Moreover, nonsynaptic interactions (diffuse neurotransmitters, gaseous messengers, neuroglia function, etc.) greatly complicate the representation of cortical integration. These arguments imply that, from an anatomical point of view, the cerebral cortex can be considered as a spatially extended system with high connectivity rather than a juxtaposition of independent areas. The level of cooperativity within and between these neuronal networks depends on the cerebral state and/or the presence of a neurological disease and would imply changes in the cerebral spatiotemporal dynamics.

The study of spatially extended systems' dynamics is still in infancy but it already provides meaningful results for the physiologists. The quantification of spatially extended systems' dynamics using time-delay embedding and correlation dimension computation has been widely tested (e.g., Lorenz, 1991). These measurements often depend on the recording site, but it has been shown that the spatial variations of correlation dimension give erroneous information about the system's dynamics. These results have been replicated in the case of EEG simulations (Lachaux et al., 1997). The validity of correlation dimension maps supposed to characterize brain dynamics is thus questionable from a strict dynamical point of view even if those indices remain statistical indicators of different states of brain activity.

Spatial embedding enables the reconstruction of spatially extended systems' dynamics, but it cannot be used to compute high correlation dimension (Lachaux et al., 1997). The computation of dimension densities (Bauer, Heng, & Martienssen, 1993), which are locally defined degrees of freedom, is an alternative approach of spatiotemporal dynamics.

Obviously, brain dynamics is spatially heterogeneous and can split into domains depicting different dynamical behaviors. Nevertheless, this fact does not mean that those domains are independent, because their very existence is a result of the interaction within the whole system. To deal with this problem, we have proposed a numerical method based on multichannel reconstruction and nonlinear forecasting that permits one to compute local complexity (i.e., entropy) for each recording site (Pezard, Martinerie, Müller, Varela, & Renault, 1996). This method leads to a set of complexity measures which defines the spatial pattern of brain dynamics' complexity (Pezard et al., 1998).

To characterize the spatial dynamical heterogeneity resulting from the interactions within a spatially extended system (such as the brain), we conjecture that local quantifiers must be defined within the context of the whole system's dynamics. Spatial embedding is the first step to take into account the whole system and quantifiers should be developed within that context.

Practical Issues

Both reconstruction methods share a set of drawbacks that are the same for all signal-processing techniques: temporal and spatial sampling, precision of the digitizer, and noisy time series of finite length. We will focus on three limiting factors: the choice of the parameters that permit one to avoid any correlation between reconstructed vector's coordinates, the issue of stationarity, and the problem of noise.

Correlation between vector's coordinates. In the case of time-delay embedding, the parameter l defines, with n, the time window $w(w = [n-1].l)$ that is a critical parameter for this method. The parameter l is a function of the temporal loss of correlation of the times series $\{x_j(t)\}$. Neither the autocorrelation function nor the mutual information are sufficient to define its optimal value (Martinerie et al., 1992). Several other methods have been proposed to define those parameters without using the trial-and-error procedure (e.g., Rosenstein, Collins, & De Luca, 1994).

In the case of spatial embedding, cross-channel correlations lead to embeddings with dependent signals. Because the electrode placement cannot be modified after the experiment to determine the best embedding, a specific procedure is needed to overcome this major drawback. SVD of the observation matrix leads to a set of linearly independent coordinates and can be used to obtain a reasonable embedding dimension (Albano et al., 1988).

The issue of time stationarity. All the embedding techniques suppose the dynamics to be time stationary during the observation epoch (several seconds for brain activity). Stationarity can be checked with recurrent plots (Eckmann, Oliffson Kamphorst, & Ruelle, 1987) or with cross-prediction techniques (Schreiber, 1997). Nevertheless, even if the stationary condition is not fulfilled, other interesting problems can be investigated. In that case, the structure of the dynamics during a defined period of time (e.g., the second following a stimulus presentation or several seconds of a particular condition) should be studied. For long periods, the variation of the dynamics' parameters certainly leads to the characterization of a family of attractors (Ruelle, 1987). For short periods, transients need to be characterized and instantaneous methods (such as wavelets) could be worth considering.

Noise. Even a low noise level (either dynamical, i.e., integrated in the dynamics, or measurement, i.e., independent from the dynamics) can cause a total loss of the dynamical structure (Casdagli, Eubank, Farmer, & Gibson, 1991). Global (Albano et al., 1988) and local (e.g., Schreiber & Grassberger, 1991) noise reduction methods have been proposed to deal with measurement noise. In the case of EEG recording, the average reference which corresponds to the sum of the signal obtained on one electrode with the average dynamics of the whole brain is a noise source easily avoided (Lachaux et al., 1997).

EEG recordings are filtered time series. The filtering is due both to the data collection apparatus and to the low pass filtering induced by passage through the skull and scalp. Filtering may induce errors in the determination of dynamics characteristics (Ott et al., 1994; Sauer et al., 1991). Simulations are needed to show exactly what information about brain dynamics can be obtained with an intrinsically filtered signal such as EEG.

Conclusion: Why Bother to Spatially Embed EEG?

Because the spatiotemporal characteristics of brain activity are important, dynamical methods for their characterization are needed. Spatial embedding, even if not the panacea, is a first attempt to deal with the spatial extension of brain dynamics. We have shown that this approach has been unfairly rejected by Pritchard et al. (1996). Furthermore several arguments suggest that time-delay method is inadequate for spatially extended systems. Spatial embedding also suffers limitations (high dimension and spatial dynamical heterogeneity for example) but a sage conclusion is to encourage a cautious development of techniques that permits local quantification within the context of global dynamics.

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